a stable foam. Column behavior for the positive systems is shown in Figures 3a and 3b.

The surface tension negative system, on the other hand, exhibited an overall column efficiency of 50%. Efficiency was increased to 56% with the addition of 0.1 to 0.4 ml of 1-decanol as shown in Figure 2. With 0.5 ml or more of surfactant, the column efficiency returned to its original value of 50%. Changing the vapor rate in the negative distillation had negligible effect on column operation and efficiency in either the presence or absence of surfactant.

Although the difference in efficiency between positive and negative systems in this study was not of the magnitude reported for the nonaqueous systems of Zuiderweg and Harmens, the difference between them was reduced by 30% with the addition of very small amounts of surfactant. The low concentrations of surfactant in the negative system increased the stability of vapor bubbles on the surface of the plates, as shown in Figures 3c and 3d and thus enhanced the interfacial area available for mass transfer. The surfactant thus caused the behavior of negative system in the sieve plate column to resemble that of the positive system, a result similar to that found in studies of supported area equipment.

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Axisymmetric Stagnation Flow Towards a Moving Plate

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In some forced convection cooling processes a coolant is impinged on a continuously moving plate. This note discusses the fluid mechanics and the heat transfer near the stagnation region.

The two-dimensional stagnation flow towards an infinite plate moving with constant velocity in its own plane was obtained by Rott (1956) and Glauert (1956). The possibility of extending the flow to three dimensions was mentioned by Rott, although this has never been done. In what follows we shall present the most important case, namely, the axisymmetric stagnation flow towards a moving plate.

AXISYMMETRIC CASE

Let the Cartesian velocity components at infinity be $u = ax$, $v = ay$, $w = -2az$ which represents a potential axisymmetric stagnation flow. Let the plate be at $z = 0$, moving with constant velocity U in the x direction.

We substitute

$$u = U\chi(\eta) + xa\varphi'(\eta) \tag{1}$$

$$v = ya\varphi'(\eta) \tag{2}$$

$$w = -2\sqrt{a\nu}\,\varphi(\eta) \tag{3}$$

$$p = -\frac{\rho}{2}\left[a^2(x^2 + y^2) + w^2 - 2\nu w_z\right] \tag{4}$$

$$\eta = \sqrt{a/\nu}\, z \tag{5}$$

into the Navier-Stokes equations and obtain

$$\varphi''' + 2\varphi\varphi'' - (\varphi')^2 + 1 = 0 \tag{6}$$

$$\varphi(0) = \varphi'(0) = 0,\ \varphi'(\infty) = 1 \tag{7}$$

$$\chi'' + 2\varphi\chi' - \chi\varphi' = 0 \tag{8}$$

$$\chi(0) = 1,\ \chi(\infty) = 0 \tag{9}$$

Equations (6) and (7) describe the well-known Homann's axisymmetric flow towards a fixed plate (1936). However Equation (8) cannot be obtained from Equation (6) through simple substitution as in the two-dimensional case. Numerical integration of the system Equations (6) to (9) is necessary. Using the Runge-Kutta method, the numerical values of χ and χ', together with more accurate values of φ and φ', are tabulated in Table 1. Figure 1 shows how χ and χ' decay with increasing distance from the plate.

The shear stress on the plate in the x direction is

$$\begin{aligned}\tau &= \mu\sqrt{\frac{a}{\nu}}\,U\left[\chi'(0) + \frac{xa}{U}\varphi''(0)\right] \\ &= \mu\sqrt{\frac{a}{\nu}}\,U\left(-0.938732 + 1.311937\frac{xa}{U}\right).\end{aligned} \tag{10}$$

It is zero at $x = 0.715531\ U/a$.

HEAT TRANSFER

Suppose the temperature of the fluid at infinity is T_∞ and the temperature of the plate is T_0. Ignoring viscous dissipation, the energy equation is

$$\rho g C_p(uT_x + vT_y + wT_z) = k(T_{xx} + T_{yy} + T_{zz}) \quad (11)$$

Substituting

$$T = T_\infty + (T_0 - T_\infty)f(\eta) \quad (12)$$

into Equation (11) gives

$$f'' + 2P\varphi f' = 0 \;\; f(0) = 1, \; f(\infty) = 0 \quad (13)$$

where P is the Peclet number $\mu C_p g/k$. Equation (13) is integrated numerically and plotted in Figure 2 for various values of P. The temperature distribution is independent of plate translation.

The heat transfer rate is

$$Q = -k(T_0 - T_\infty)\sqrt{\frac{a}{\nu}}f'(0) \quad (14)$$

We see the heat transfer is proportional to $\sqrt{a}$ or to the square root of the magnitude of the normal velocity. The derivative $f'(0)$, however, is quite dependent on the Peclet number (Table 2).

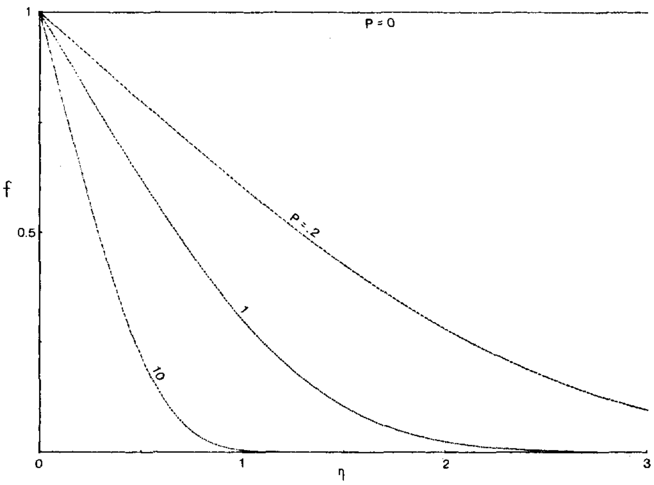

Fig. 2. Temperature distribution.

NOTATION

a = constant multiplicative factor of the flow at infinity, of dimensions, 1/time

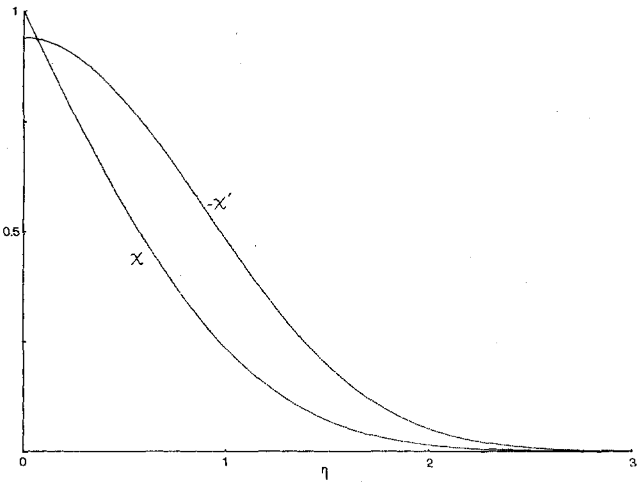

Fig. 1. Velocity distribution due to the moving plate.

TABLE 1. NUMERICAL VALUES

η	φ	φ'	χ	χ'
0	0	0	1.00000	−0.938732
0.1	0.00639	0.12619	0.90634	−0.93233
0.2	0.02490	0.24239	0.81393	−0.91379
0.3	0.05454	0.34863	0.72395	−0.88412
0.4	0.09430	0.44498	0.63744	−0.84442
0.5	0.14321	0.53160	0.55535	−0.79599
0.6	0.20030	0.60870	0.47849	−0.74030
0.7	0.26464	0.67662	0.40748	−0.67900
0.8	0.33534	0.73577	0.34281	−0.61389
0.9	0.41152	0.78665	0.28477	−0.54684
1.0	0.49241	0.82986	0.23345	−0.47968
1.2	0.66542	0.89597	0.15051	−0.35182
1.4	0.84932	0.93982	0.09154	−0.24139
1.6	1.0402	0.96717	0.05237	−0.15451
1.8	1.2354	0.98315	0.02810	−0.09206
2.0	1.4330	0.99188	0.01411	−0.05098
2.5	1.9312	0.99902	0.00187	−0.00838
3.0	2.4311	0.99992	0.00016	−0.00085
3.5	2.9310	0.99999	0.00001	−0.00005
4.0	3.4310	0.99999	0.00000	−0.00000

TABLE 2. DEPENDENCE OF HEAT TRANSFER ON PECLET NUMBER

P	$f'(0)$
0	0
0.2	−0.40384
1	−0.76223
5	−1.37221
10	−1.75204
∞	−∞

C_p = specific heat at constant pressure
f = normalized temperature profile
g = gravitation constant
k = thermal conductivity
p = pressure
P = Peclet number
Q = heat transfer per area per time
T = temperature
T_0 = temperature of plate
T_∞ = temperature of fluid at infinity
u = velocity component in x direction
U = velocity of plate
v = velocity component in y direction
w = velocity component in z direction
x = cartesian coordinate
y = cartesian coordinate
z = cartesian coordinate

Greek Letters

η = normalized distance from plate
μ = absolute viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress
φ = Homann's velocity variable
χ = velocity profile due to translation

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